**Genetic programming-based backbone curve model of reinforced concrete walls**

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**Abstract:**

Backbone curve, as a nonlinear response analysis method, can be used for performance assessment of residual resistance and performance prediction during preliminary design of structures. In this study, a backbone curve model of reinforced concrete (RC) walls based on Genetic programming-based symbolic regression (GP-SR) was proposed, which can help to quickly evaluate the bearing capacity and seismic performance of RC walls. Unlike the black-box characteristic of traditional machine learning models, the GP-SR method can give explicit computational equations, which is more interpretable and easier to be used by researchers and engineers. Experimental data of 388 existing RC walls were used for feature selection, model training, and comparison with the modeling method of ASCE 41-17 to verify its effectiveness for modeling the backbone curves of RC walls with four failure modes (i.e., flexure, flexure-shear, shear, and shear-sliding). The results showed that the accuracy of GP-SR model was better than that of ASCE 41-17 except for flexural yield load and peak load. However, for the flexural yield load and peak load, the prediction results of the GP-SR model were also very close to those of ASCE 41-17. Overall, the GP-SR model described well the backbone curves of RC walls with various design conditions.

**Keywords:** reinforced concrete wall; machine learning; SHAP; symbolic regression; backbone curve

**1 Introduction**

Shear walls have been widely used in modern high-rise buildings as lateral force resisting members. Among the various types of shear walls, reinforced concrete (RC) shear walls are the most widely used, so that the analysis of the nonlinear response of such structures is of great significance for the seismic design and evaluation of building structures. A number of researchers have already analyzed the nonlinear response of RC shear walls by means of finite element analysis, including the development of refined modeling methods. The most common modeling methods include fiber models [1], multi-vertical rod models [2], and layered shell models [3]. However, the finite element modeling approach is usually very time-consuming and is not conducive to the performance prediction during the initial design of structures and the rapid assessment of the residual seismic performance of the structures after earthquake damage.

FEMA 356 [4] proposed a backbone curve analysis model for RC shear walls to evaluate the nonlinear response of shear walls governed by flexure and shear. The RC shear wall backbone curve model adopted in ASCE 41-17 [5] (**Fig. 1**) was modified from the model specified in FEMA 356 [4] to supplement the definition of cracking points for shear-controlled shear walls, where the ratio of cracking load to yield load is a certain value (). For the calculation of the displacement of each characteristic point on the backbone curve, ASCE 41-17 [5] provides two tables (i.e., Tables 10-19 and 10-20 in Chapter 10) for the interpolation of the flexure-controlled and shear-controlled walls, respectively.

|  |  |
| --- | --- |
|  |  |
| 1. shear-controlled | 1. flexure-controlled |
| **Fig. 1. Force-deformation relationships for concrete structure walls** | |

The backbone curve model specified in ASCE 41-17 [5] considers only two cases (i.e., flexure-controlled and shear-controlled), and the two failure modes (i.e., shear sliding and flexure-shear failure) are not explicitly analyzed. Meanwhile, the calculation table of ASCE 41-17 [5] is relatively simplified for the modeling parameters considered. In the shear-controlled walls, the drift ratio at each characteristic point is defined by only the axial load ratio. In the flexure -controlled walls, the drift ratio at each characteristic point is determined from the axial load ratio, normalized shear stress, and confinement of boundary members. Linear interpolation between the modeling parameters is considered to determine the drift ratio.

It can be seen that the ASCE 41-17 [5] model does not discuss all failure modes, while no definite formula is given for the backbone curve modeling parameters, and the design parameters considered are very limited. Due to such obvious limitations of the backbone curve model in ASCE 41-17 [5], a new backbone curve model needs to be developed to analyze the nonlinear response of RC shear walls, which provides definitive formulas to more accurately describe backbone curves. Further, the backbone curve model should be simplified enough to facilitate engineering applications. Allouzi et al. [6] developed a set of empirical equations to define the backbone curve model using 117 existing RC wall test results. However, unlike actual situation, the yield load was simply assumed to be the same to the peak load of the shear walls regardless of failure modes. Epackachi et al. [7] proposed an empirical equation for the peak strength of the backbone curve of shear-controlled walls specified in ASCE 41-17 [5] based on 240 existing test data, and analyzed the effect of design parameters on the displacement at characteristic points. However, fixed values were still used to define the displacement of characteristic point. Weng et al. [8] proposed a trilinear backbone curve model for squat walls based on softened strut-and-tie model, and each characteristic point was defined as equations. However, the drift ratio of the backbone curve was still slightly overestimated, and the model was only applicable to shear-controlled walls.

As such, some progress has been made in the development of backbone curve models for RC shear walls, but there are still many limitations. Current empirical or semi-empirical backbone curve models either adopt too many simplified assumptions or show low prediction accuracy for the characteristic points, which is related to a priori knowledge for empirical equations. In general, the development of empirical or semi-empirical equations requires researchers to have considerable knowledge of the domain to combine the design parameters into polynomials through some theoretical analysis before performing linear regression. Once researchers have insufficient expert experience, they usually rely on correlation analysis and linear regression to establish empirical equations [6], but linear regression does not describe the nonlinear relationships accurately. For this reason, it is difficult to build empirical equations for complex nonlinear relationships.

In recent years, machine learning has been widely introduced as an emerging technique in the field of structural seismic performance analysis and used to predict nonlinear responses. Zhang et al. [9] used machine learning methods to predict the failure mode, shear strength, and deformation capacity of RC walls. Nguyen et al. [10] used artificial neural networks (ANN) to predict the shear strength of RC flanged dwarf walls. Mangalathu et al. [11] used various machine learning algorithms to predict the failure mode of RC shear walls, showing the best prediction accuracy of 86% for random forest method. The backbone curve model based on machine learning is defined as a multi-output regression problem. Luo and Paal [12] used a multi-output support vector machine model to estimate the backbone curve of RC columns. Deger and Taskin [13] used a multi-output Gaussian process regression model to estimate the backbone curve of RC walls. However, the existing studies used black box models to build predictive models, and it is difficult for users to explain the complex relationships between input and output variables from the models themselves. Although Feng et al. [16] and Mangalathu et al. [17] applied SHapley Additive exPlanations (SHAP) to quantitatively explain the effect of each input variable on the predicted outcome, it still did not change the nature of the black-box model (unable to describe the functional relationship between input and output variables through explicit mathematical equations). Unlike the conventional machine learning algorithms, genetic programming-based symbolic regression (GP-SR) is a white-box model [14], which has the advantage of being able to obtain functional relationships between inputs and outputs without pre-assuming a functional form. GP-SR was used to predict the shear strength of RC beams [15] and shear strength of steel fiber reinforced concrete beams [16].

In the present study, GP-SR was applied to model the backbone curves of RC walls with various failure modes, and to propose explicit functional relationships. Addressing the problem that the failure mechanisms of RC walls were dependent on failure modes, the backbone curves were not generated by building a unified model, but the failure modes of RC walls were discussed according to their classification. Due to the poor generalization performance of GP-SR in high-dimensional problems [17], a SHAP-based feature selection method was also adopted. To verify the validity of the proposed model, the GP-SR model was compared with the model specified in ASCE 41-17. The results showed that the proposed model predicted better the test results of RC walls for all output variables regardless of failure modes, except and of RC walls governed by flexural failure. The predictions of the GP-SR model for and of RC walls governed by flexural failure were also very close to those of ASCE 41-17 [5].

**2 ASCE41-17 model description**

In ASCE 41-17 [5], assuming , the peak load of RC walls at each failure mode is estimated according to ACI 318-19 [51]. For flexure-shear controlled walls, the smaller of the flexural strength and shear strength is selected as the peak load.

***Flexural failure mode***

The shear strength (*Vp*) of the RC walls governed by flexure failure is estimated based on the nominal flexural moment, as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (27) |

where is the nominal flexural moment calculated based on strain coordination when the concrete strain reaches 0.003; and is the height of the shear wall.

***Shear failure mode***

The shear strength (*Vp*) of the RC walls governed by shear failure is estimated as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (28) |

where is the cross-sectional area of the RC wall; is the coefficient related to the wall shape (= 0.25 for , 0.17 for , and linear interpolation between 0.17 and 0.25 for ); is the reinforcement ratio of transverse reinforcement in the web; is the yield strength of transverse reinforcement in the web; and is the concrete strength.

***Shear sliding failure*** ***mode***

The shear strength (*Vp*) of the RC walls governed by shear sliding failure is estimated as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (29) |

where is the cross-sectional area of the reinforcement passing through the shear plane; is the yield strength of the reinforcement; and μ is the friction coefficient (= 1.4 for normal weight and integrally placed concrete).

ASCE 41-17 [5] specifies the modeling parameters for the backbone curve of RC walls governed by flexure failure (**Table 1**) and shear failure (**Table 2**) (refer to **Fig. 1**). Since the modeling parameters for shear sliding and flexure-shear controlled RC walls were not explicitly analyzed in ASCE 41-17 [5], in this study, the parameters for shear-controlled walls were used for the RC walls governed by shear sliding failure. For flexure-shear controlled RC walls, the shear strength and flexural strength were used to determine whether the walls were controlled by shear or flexure, and the corresponding modeling parameters were used.

**Table 1** Modeling parameters for RC walls controlled by flexure

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Confined Boundary |  |  |  |
|  |  | Yes | 0.005 | 0.015 | 0.20 |
|  |  | Yes | 0.004 | 0.010 | 0.015 |
|  |  | Yes | 0.003 | 0.009 | 0.012 |
|  |  | Yes | 0.0015 | 0.005 | 0.010 |
|  |  | No | 0.002 | 0.008 | 0.015 |
|  |  | No | 0.002 | 0.006 | 0.010 |
|  |  | No | 0.001 | 0.003 | 0.005 |
|  |  | No | 0.001 | 0.002 | 0.004 |

**Table 2** Modeling parameters for RC walls controlled by shear

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  | 0.004 | 0.01 | 0.02 |
|  | 0.004 | 0.0075 | 0.01 |

**2 Database description**

To develop a prediction model for backbone curves of RC shear walls with various failure modes, a database containing 388 existing RC shear wall experimental data was collected, of which 212 experimental data were taken from an existing database [18] and 176 experimental data were collected by the authors [19–48]. Note that the RC shear walls repaired or retrofitted, and RC shear walls with web diagonal reinforcement or composite materials were excluded in the database. Further, for simple design model, only symmetric section (i.e., rectangular section for 279 walls, and barbell or flange section for 109 walls) was considered in the database. Asymmetric section and hollow section were not discussed in this study. The failure modes of the RC shear walls were governed by shear sliding failure for 26 walls, shear failure for 144 walls, flexure-shear failure for 84 walls, and flexure failure for 134 walls.

The design parameters of the existing RC shear walls in the database includes the height (), wall width (), web thickness (), boundary member length (), boundary member thickness (), cross-sectional area () (refer to **Fig. 2**), concrete strength (), rebar yield strength (, , , and ), reinforcement ratios of longitudinal and transverse reinforcement in the web ( and ), reinforcement ratios of longitudinal and transverse reinforcement in the boundary members ( and ), axial load ratio (), and failure mode.

|  |
| --- |
| 图片2 |
| **Fig. 2**. Schematic diagram of the geometric parameters of the wall section |

***2.1 Definition of backbone curve model and output variables***

As shown in **Fig. 3**, the backbone curve model proposed in this study consists of three control points including the yield point, peak point, and ultimate point. At the yield point ( and ), the yield stiffness is defined as the secant stiffness passing through the point at 75% of the peak load on the backbone curve [49]. The yield displacement is determined from the peak load divided by the yield stiffness (**Fig. 3(b)**). The yield load is defined as the corresponding point on the backbone curve at . The peak point ( and ) is defined as the point on the backbone curve at the peak load . The ultimate point ( and ) corresponds to the state when the load-carrying capacity of the specimen decreases to 80% of the peak load. The output variables consist of , , , , and .

|  |  |  |
| --- | --- | --- |
| 图片1 | 图片b | 图片c |
| (a) Hysteresis curve of RCW1 [50] | (b) Experimental backbone curve | (c) Proposed backbone curve |
| **Fig. 3**. Schematic diagram of the proposed backbone curve model in a typical specimen | | |

***2.2 Input variables***

From the available studies[12,13], machine learning based backbone curve models can be regarded as multiple output regression problems, in which the same input variables are used to predict multiple output variables. For the backbone curve model, two dimensions of output variables exist. Thus, use of the same input variables would result in the problem of incongruent model dimensions and reduced interpretability. To consider issues of dimensional coordination and interpretability, on the basis of empirical equations specified in current design guideline and code [5,51], two sets of input variables were designed for the output variables (i.e., displacement and load), which includes the geometry, reinforcement details, and axial load (**Table 3**). **Fig.** **4** shows the distribution of all input and output variables in the database used in this study.

**Table 3** Input variables

|  |  |  |
| --- | --- | --- |
| Types | Load input variables | Displacement input variables |
| Geometric configuration | / | / |
| Reinforcement detail |  | /  /  /  / |
| Axial load |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| H | Sg | Hlw | lwbw |
| hblw | bbw | fc | cvfy |
| chfy | wvfy | whfy | cvfyfc |
| chfyfc | wvfyfc | whfyfc | n |
| dis_yeild | load_yeild | dis_peak | load_peak |
| dis_max | | | |
| **Fig. 4**. Distribution of input and output parameters | | | |

**3 SHAP-based feature selection**

***3.1 Shapley additive explanations (SHAP)***

Lundberg et al. [52] proposed SHAP for interpreting machine learning models to quantify the effect of the feature parameters on the final output value of the model. As a uniformly interpretable method, SHAP has become a feature selection method in recent years because the importance of input variables can be assessed by calculating their SHAP values [53]. The interpreted form of SHAP is expressed as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where is the initial machine learning model; is the corresponding explanation model; is the input variables of ;is the simplified input variable with a mapping relationship to *x*; *M* is the number of input variables; is the output value when all input variables are 0; and is the Shapley value of the *k*th input variable.

***3.2 Feature selection methods***

The SHAP-based feature selection method is shown in **Fig. 5**. The initial machine learning model was built based on a training set and a test set, in which the training set and test set were divided randomly in a ratio of 7:3. The training set was used to train the model, and the test set was used to evaluate the performance of the model. Five machine learning models were used to predict the characteristic points of the backbone curve of RC shear walls showing four failure modes, and the best performing model was selected. The five machine learning models used are decision trees (DT), random forests (RF), gradient boosting trees (XGBoost), support vector machines (SVM), and artificial neural networks (ANN). The prediction performance of the five machine learning models in the test set was evaluated by (**Eq. (2)**) and (**Eq. (3)**) (refer to **Table 4**). Of the five machine learning models used, XGBoost exhibited the best prediction results (for all output variables, is above 0.75), and the tree-based models in DT, RF, and XGBoost were better than SVM and ANN. XGBoost showing the best prediction performance was chosen for SHAP analysis. In SHAP analysis, the features are sorted by the mean of absolute SHAP values (i.e., mean(|SHAP value|)), and the **n** highest ranked features are selected. It is noted that **n** is not fixed, but is defined by a threshold value related to mean(|SHAP value|). The threshold value implies 90% of the total mean(|SHAP value|).

|  |
| --- |
|  |
| **Fig. 5**. Feature selection process based on SHAP |

**Table 4** Predictive performance of machine learning models in various failure modes

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Output | Failure mode | Model | | | | | | | | | |
| SVM | | ANN | | DT | | RF | | XGBoost | |
|  |  |  |  |  |  |  |  |  |  |
|  | F | 0.64 | 3.50 | 0.51 | 4.10 | 0.77 | 2.82 | 0.69 | 3.27 | **0.78** | **2.72** |
| FS | 0.61 | 3.14 | 0.70 | 2.75 | 0.66 | 2.97 | 0.75 | 2.48 | **0.76** | **2.44** |
| S | 0.89 | 3.21 | 0.89 | 3.24 | 0.95 | 2.27 | 0.94 | 2.37 | **0.97** | **1.78** |
| SL | 0.92 | 1.81 | 0.92 | 1.91 | 0.92 | 1.90 | 0.87 | 2.41 | **0.93** | **1.79** |
|  | F | 0.78 | 80.55 | 0.81 | 59.72 | 0.86 | 50.38 | 0.72 | 72.47 | **0.87** | **49.87** |
| FS | 0.90 | 80.95 | 0.91 | 74.91 | 0.85 | 96.29 | 0.83 | 103.36 | **0.93** | **66.49** |
| S | 0.86 | 209.07 | 0.87 | 200.33 | 0.87 | 201.14 | 0.89 | 181.95 | **0.93** | **141.1** |
| SL | 0.85 | 192.6 | 0.83 | 200.31 | 0.86 | 181.65 | 0.81 | 214.74 | **0.90** | **156.81** |
|  | F | 0.74 | 13.40 | 0.80 | 11.74 | 0.84 | 10.25 | 0.77 | 12.66 | **0.86** | **9.86** |
| FS | 0.70 | 7.64 | 0.81 | 6.13 | 0.71 | 7.62 | 0.71 | 7.51 | **0.81** | **6.08** |
| S | 0.85 | 12.22 | 0.90 | 10.03 | 0.95 | 6.81 | 0.93 | 8.65 | **0.96** | **6.70** |
| SL | 0.82 | 5.13 | 0.86 | 4.56 | 0.62 | 7.48 | 0.92 | 3.32 | **0.94** | **2.89** |
|  | F | 0.76 | 77.42 | 0.80 | 71.30 | 0.78 | 75.20 | 0.74 | 80.53 | **0.87** | **58.29** |
| FS | 0.90 | 80.95 | 0.91 | 74.91 | 0.85 | 96.29 | 0.83 | 103.36 | **0.92** | **68.86** |
| S | 0.90 | 198.21 | 0.88 | 222.40 | 0.87 | 227.41 | 0.87 | 233.45 | **0.92** | **177.49** |
| SL | 0.87 | 237.91 | 0.86 | 243.16 | 0.73 | 341.46 | 0.82 | 276.03 | **0.91** | **193.54** |
|  | F | 0.82 | 14.76 | 0.80 | 15.46 | 0.88 | 11.99 | 0.82 | 14.65 | **0.92** | **9.95** |
| FS | 0.67 | 9.83 | 0.73 | 8.87 | 0.67 | 9.74 | 0.69 | 9.50 | **0.82** | **7.25** |
| S | 0.79 | 15.79 | 0.87 | 12.14 | 0.93 | 8.77 | 0.93 | 9.00 | **0.94** | **8.12** |
| SL | 0.88 | 8.39 | 0.81 | 10.69 | 0.86 | 9.07 | 0.81 | 10.65 | **0.97** | **4.22** |

Note: F denotes flexure failure; FS denotes flexure-shear failure; S denotes shear failure; and SL denotes shear-sliding failure.

***3.3 Model interpretation and feature selection results***

The importance ranking of the input variables for shear sliding controlled RC walls are shown in **Fig.** **6**. In this figure, the red boxed sections represent the selected input variables. In the case of , the key input variables were , , , and . In the case of , the key input variables were, , , and . In the case of , the key input variables were, , , and . In the case of , the key input variables were /, , , , , , and . In the case of , the key input variables were /, , , , , and . These results indicate that the input variables (, , , and ) significantly affect the loads and displacements in the RC walls governed by shear sliding failure. Further, the loads are significantly affected by / that would be related to friction, which can be explained in terms of the failure mechanism of shear sliding.

Grammatikou et al. [18] describes the shear sliding failure mechanism, as follows. When flexural yielding occurs at the wall, a full-length crack is developed in the base section. As the moment increases further, the compression bars yield, and the crack at the tension zone is closed. During the early sliding before the crack at the tension zone is closed, the shear sliding resistance of the wall is mainly provided by the dowel action of the vertical reinforcement. When the wall edge crack is closed, the concrete in the compression zone due to the axial load and moment generates friction. Thus, the shear sliding resistance of the wall is developed by both the friction in the compression zone and the dowel action of the vertical reinforcement. For the section configuration (/), flange and barbell shaped walls with larger / generate the larger compression zone area and flexural strength than rectangular walls, which decreases the concrete damage in the compression zone before the reinforcement yields and ensures the stability of the friction in the compression zone. and also affect the friction in the compression zone. The vertical reinforcement ( and ) is directly related to the dowel action.

|  |  |  |  |
| --- | --- | --- | --- |
| dis_yeild | dis_peak | | dis_max |
|  |  | |  |
| load_yeild | | load_peak | |
| (d) | | (e) | |
| **Fig. 6.** Importance ranking of input variables for shear sliding controlled RC walls | | | |

The importance ranking of the input variables for shear-controlled walls are shown in **Fig. 7**. In the case of , the key input variables were , , , , , and . In the case of , the key input variables were , , , , , and . In the case of , the key input variables were , , , , , , and . In the case of , the key input variables were , , , , /, , and . In the case of , the key input variables were , , ,/, , and . These results indicate that the vertical distribution reinforcement () and geometry shape (, , /, and ) significantly affect the displacements in the RC walls governed by shear failure. Further, significantly affects , as demonstrated in a previous study [37]. Some input variables (, , /, and ) significantly affect the loads, which is consistent with the experimental results [37,54].

|  |  |  |  |
| --- | --- | --- | --- |
| dis_yeild | dis_peak | | dis_max |
| (a) | (b) | | (c) |
| load_yeild | | load_peak | |
| (e) | | (f) | |
| **Fig. 7.** Importance ranking of input variables for shear controlled RC walls | | | |

The importance ranking of the input variables for flexure-shear controlled RC walls are shown in **Fig.** **8**. In the case of , the key input variables were , , , , , , and In the case of , the key input variables were , , , , , and . In the case of , the key input variables were , , , , , , and . In the case of , the key input variables were , , , , /, , , and . In the case of , the key input variables were , , , /, , , and . From the SHAP analysis results, for flexure-shear controlled RC walls, the results indicate that the input variables (, , , , , and ) have a significant effect on the displacements. Some input variables (, , /, , , and ) have a significant effect on the loads.

|  |  |  |  |
| --- | --- | --- | --- |
| dis_yeild | dis_peak | | dis_max |
| (a) | (b) | | (c) |
| load_yeild | | load_peak | |
| (e) | | (f) | |
| **Fig. 8.** Importance ranking of input variables for flexure-shear controlled RC walls | | | |

The importance ranking of the input variables for flexure-controlled walls are shown in **Fig.** **9**. In the case of , the key input variables were , , , , , and . In the case of , the key input variables were , , , , , and . In the case of , the key input variables were , , , , , , and . In the case of , the key input variables were , , , , , , , and . In the case of , the key input variables were , , , , , , and . From the SHAP analysis results, for flexure RC walls, has a very significant effect on because flexural yielding usually represents the yielding of the tension reinforcement at the edge of the wall section.has a very significant effect on because the flexure failure occurs due to damage at the base of the boundary element, and the concrete confinement by the transverse reinforcement in the boundary element can significantly improve the flexural ductility [55,56].

In summary, the relationships between the input and output variables are affected by failure modes (or failure mechanisms) of RC walls. Thus, it is necessary to model the backbone curves of RC walls individually, addressing each failure mode.

|  |  |  |  |
| --- | --- | --- | --- |
| dis_yeild | dis_peak | | dis_max |
| (a) | (b) | | (c) |
| load_yeild | | load_peak | |
| (e) | | (f) | |
| Fig. 9. Importance ranking of input variables for flexure-controlled RC walls | | | |

**4 Regression model for backbone curve of RC shear wall**

***4.1 Statistical metrics***

Statistical metrics are mainly used to measure the model prediction performance and build the fitness function in constructing genetic programming. The indicators applied in this study are the coefficient of determination (), Root Mean Square Error (*RMSE*), mean (), standard deviation (), and coefficient variation (*COV*). All metrics are defined by experimental value () and predicted value (), and the metrics are defined as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |
|  |  | (3) |
|  |  | (4) |
|  |  | (5) |
|  |  | (6) |

***4.2 Symbolic regression based on genetic programming***

Genetic programming-based symbolic regression (GP-SR) is a genetic algorithm-based regression method proposed by Koza [14] for establishing a functional relationship between input and output variables while minimizing error indicators. Davidson et al. [57] introduced least squares based on the method of Koza [14] to improve the ability of the algorithm to search for constant terms in the symbolic model. Symbolic regression differs from common regression methods: 1) Symbolic regression does not require a priori knowledge to assume a functional form of the regression equation; 2) However, the selection, crossover, and variation steps of the genetic algorithm are used to build the regression model. The GP-SR starts with a random combination of operational symbols (e.g., addition, subtraction, division, multiplication, etc.), input variables, and constants to generate a tree structured symbolic model for building the initial population. The selected tree expressions are crossed over and mutated for recombination to generate the next generation of populations, using the fitness function to select the best individuals in each generation of populations. As shown in **Fig.** **10(a)**, crossover implies that the subtrees of two pairs of tree-like expressions are cross-swapped, and a new expression is generated. The mutations include point mutation, subtree mutation, and hoist mutation. The point mutation and subtree mutation increase the variety within populations to better search for the best model (**Fig.** **10(b)** and **(c)**). The hoist mutation suppresses the expansion of the tree expression, which helps to obtain a concise model (**Fig.** **10(d)**). The algorithm repeats this process until it reaches a set maximum number of generations and/or generates the model with the best current performance. All programs in this study were developed in python. The GP-SR model relies on the open source project gplearn [58], and the list of parameters used is shown in **Table 5**.

|  |  |
| --- | --- |
| 交叉示意图 | 亚树突变 |
| 1. Crossover | 1. Subtree mutation |
| 点变异 | 葫芦突变 |
| 1. Point mutation | 1. Hoist mutation |
| Fig. 10. Operations in GP-SR model | |

**Table 5** Model parameters used in the GP-SR

|  |  |
| --- | --- |
| Parameters | Values |
| Population size | 2000,3000,4000,5000 |
| Initial tree depth | (2,6) |
| Initial method | grow, full, half and half |
| Crossover rate | 0.5, 0.6, 0.7 |
| Subtree mutation rate | 0.2, 0.11, 0.1 |
| Point mutation rate | 0.1, 0.2 |
| Hoist mutation rate | 0.1, 0.2 |
| Parsimony coefficient | 0.001, 0.002, 0.003, 0.5 |
| Function set | add, sub, mul, sqrt, inv |
| Metric | , *RMSE* |

**5 Proposed model**

The backbone curves of RC walls with various failure modes were modelled separately based on GP-SR. The input variables to train models were filtered by SHAP analysis. When screening the models obtained from the GP-SR, model complexity was considered in addition to statistical metrics. The models with equation lengths greater than 40 was directly excluded. For simple models, apart from manual filtering, some model parameters were tried to change. In general, the higher hoist mutation rate and parsimony coefficient are better to control the model complexity. However, at the same time, it is possible to inhibit the variety of population which leads to poor predictive performance of the final model. Thus, during the training process, the model parameters need to be adjusted according to the model complexity and statistical metrics. It is noted that the proposed model requires to determine the failure mode in advance. Some data-driven methods are recommended [11], which can accurately predict the failure mode of RC shear walls.

***5.1 Yield point***

The yield displacement () and yield load () are calculated from **Eqs. (7)**-**(10)** and **Eqs.** **(11)-(14)**, respectively, according to failure modes. It is noted that in the equations, the unit of displacement output variables and load output variables are mm and kN, respectively. The predictions of and in the training and test sets are presented in **Figs.** **11** and **12**, respectively. The values of at in test set (i.e., = 0.84 for sliding shear failure, = 0.91 for shear failure, = 0.68 for flexure-shear failure, and = 0.58 for flexure failure) indicate that the proposed models for predicts well the test results of RC walls showing shear sliding failure and shear failure, but the dispersion of the predictions for RC walls showing flexure failure is relatively larger (*COV*= 0.47). The values of at in test set (i.e., = 0.87 for sliding shear failure, = 0.87 for shear failure, = 0.93 for flexure-shear failure, and = 0.86 for flexure failure) indicate that the proposed models for predicts well the test results of RC walls regardless of failure modes.

|  |  |
| --- | --- |
|  | (7) |
|  | (8) |
|  | (9) |
|  | (10) |
|  | (11) |
|  | (12) |
|  | (13) |
|  | (14) |

|  |  |
| --- | --- |
| F_dis_yeild | FS_dis_yeild |
| (a) Flexure failure | (b) Flexure-shear failure |
| S_dis_yeild | SL_dis_yeild |
| (c) Shear failure | (d) Shear sliding failure |
| Fig.11.Comparison between the prediction results and test results of in each failure mode | |

|  |  |
| --- | --- |
| F_force_yeild | FS_force_yeild |
| (a) Flexure failure | (b) Flexure-shear failure |
| S_force_yeild | SL_dis_yeild |
| (c) Shear failure | (d) Shear sliding failure |
| Fig.12.Comparison between the prediction results and test results of in each failure mode | |

***5.2 Peak point***

The peak displacement () and peak load () are caculated from **Eqs. (15)-(18)** and **Eqs. (19)-(22)**, respectively, according to failure modes. The predictions of and in the training and test sets are presented in **Figs.** **13** and **14**, respectively. The values of at (i.e., =0.80 for sliding shear failure, =0.87 for shear failure, =0.80 for flexure-shear failure, and =0.80 for flexure failure) and at (=0.90 for sliding shear failure, =0.89 for shear failure, =0.89 for flexure-shear failure, and =0.85 for flexure failure) in test set indicate that the proposed models for and predict well the test result of each failure mode.

|  |  |
| --- | --- |
|  | (15) |
|  | (16) |
|  | (17) |
|  | (18) |
|  | (19) |
|  | (20) |
|  | (21) |
|  | (22) |

|  |  |
| --- | --- |
|  |  |
| (a) Flexure failure | (b) Flexure-shear failure |
|  |  |
| (c) Shear failure | (d) Shear sliding failure |
| Fig.13. Comparison between the prediction results and test results of in each failure mode | |

|  |  |
| --- | --- |
|  |  |
| (a) Flexure failure | (b) Flexure-shear failure |
|  |  |
| (c) Shear failure | (d) Shear sliding failure |
| Fig.14.Comparison between the prediction results and test results of in each failure mode | |

***5.3 Ultimate point***

The ultimate point displacement ( is calculated from **Eqs. (23)-(26)** according to failure modes. The predictions of in the training and test sets are presented in **Fig.** **15**. The values of at (=0.86 for sliding shear failure, =0.80 for shear failure,=0.86 for flexure-shear failure and =0.79 for flexure failure) indicate that the proposed models for predicts well the test results of each failure mode.

|  |  |
| --- | --- |
|  | (23) |
|  | (24) |
|  | (25) |
|  | (26) |

|  |  |
| --- | --- |
|  |  |
| (a) Flexure failure | (b) Flexure-shear failure |
|  |  |
| (c) Shear failure | (d) Shear sliding failure |
| **Fig.15.** Comparison between the prediction results and test results of in each failure mode | |

**6 Results and discussion**

To verify the validity of the proposed models, the GP-SR model was compared with the model specified in ASCE 41-17 [5]. The comparison results for the test set are shown in **Table 6**, **Fig.** **16**, and **Fig.** **17**. The predictions of the GP-SR model were better than those of ASCE 41-17 [5], showing the mean value close to 1.0, except for ~~and~~  of RC walls governed by flexural failure. However, the predictions of the GP-SR model for ~~and~~  of RC walls governed by flexural failure were also very close to those of ASCE 41-17 [5]. Except for RC walls governed by shear sliding failure, ASCE 41-17 [5] predicted the load output variables better than the displacement output variables. This is because ASCE 41-17 [5] specifies the displacements using table interpolation calculations that brings unknown dispersion. As seen in **Table 6**, ASCE 41-17 [5] significantly overestimated and of RC walls governed by shear sliding failure and of RC walls governed by shear failure. On the other hand, ASCE 41-17 [5] exhibited acceptable prediction performance in other cases (in particular, at the load output variables). ~~In the proposed GP-SR model, the prediction of was significantly affected by failure modes. The proposed model predicted significantly better the test results of RC walls governed by shear failure and shear sliding failure than those of RC walls governed by flexure failure and flexure-shear failure. Thus, of RC walls failed in flexure needs to be carefully estimated in the proposed model. How? of RC walls failed in flexure may be not estimated very exactly in the proposed model.~~

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | | |  |
|  | | |  | |
| **Fig.16.** Predictions of ASCE 41-17 for each output variable in the test set | | | | |
|  |  | | |  |
|  | |  | | |
| **Fig.17.** Predictions of GP-SR for each output variable in the test set | | | | |

**Table 6** Comparison between GP-SR model and ASCE 41-17 in the test set

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Output | Failure mode | GP-SR model | | | | ASCE 41-17 | | | |
|  |  |  | *COV* |  |  |  | *COV* |
|  | F | **0.55** | **3.03** | **0.95** | **0.47** | 0.13 | 4.23 | 0.87 | 0.54 |
| FS | **0.68** | **2.83** | **1.03** | **0.33** | 0.21 | 4.47 | 0.72 | 0.44 |
| S | **0.91** | **2.93** | **1.00** | **0.35** | 0.62 | 5.96 | 1.12 | 0.46 |
| SL | **0.84** | **2.66** | **0.92** | **0.39** | 0.37 | 5.20 | 1.24 | 0.56 |
|  | F | 0.86 | 63.67 | 1.05 | 0.29 | **0.93** | **46.05** | **1.07** | **0.24** |
| FS | **0.93** | **66.53** | **0.92** | **0.30** | 0.74 | 127.17 | 0.95 | 0.48 |
| S | **0.87** | **197.47** | **1.03** | **0.37** | 0.72 | 290.11 | 1.13 | 0.42 |
| SL | **0.87** | **176.46** | **1.05** | **0.35** | -0.28 | 552.84 | 2.34 | 0.33 |
|  | F | **0.80** | **11.90** | **0.86** | **0.49** | -0.04 | 26.90 | 0.93 | 0.55 |
| FS | **0.80** | **6.16** | **1.04** | **0.33** | 0.28 | 11.89 | 1.30 | 0.36 |
| S | **0.87** | **11.54** | **1.16** | **0.47** | 0.45 | 23.57 | 1.27 | 0.60 |
| SL | **0.80** | **5.35** | **0.91** | **0.34** | 0.26 | 10.34 | 1.06 | 0.52 |
|  | F | 0.85 | 75.08 | 1.10 | 0.24 | **0.88** | **69.40** | **0.88** | **0.24** |
| FS | **0.89** | **101.62** | **1.04** | **0.28** | 0.68 | 171.38 | 0.81 | 0.48 |
| S | **0.89** | **215.26** | **0.98** | **0.37** | 0.69 | 353.99 | 0.97 | 0.42 |
| SL | **0.89** | **214.10** | **1.00** | **0.38** | 0.64 | 391.18 | 2.00 | 0.34 |
|  | F | **0.79** | **15.72** | **1.10** | **0.59** | 0.14 | 31.95 | 1.22 | 0.71 |
| FS | **0.87** | **6.16** | **1.12** | **0.20** | 0.36 | 13.59 | 1.18 | 0.47 |
| S | **0.80** | **15.30** | **1.01** | **0.59** | 0.63 | 20.96 | 1.84 | 0.78 |
| SL | **0.86** | **9.09** | **0.97** | **0.37** | 0.15 | 22.61 | 1.45 | 0.60 |

A parametric study showing the relationship between input variables and output variables is needed. As equations are complicated, figures showing a tendency of output variables at each parameter would be useful to be understood. Also, in the same figure, ASCE model should also be included for direct comparison between two models.

**7 Conclusion**

In the present study, a backbone curve model for RC walls was proposed based on GP-SR. The displacements and loads corresponding to the yield, peak, and ultimate points were modeled, addressing four failure modes including shear sliding, shear, flexure-shear, and flexure. To avoid poor generalization performance at high-dimensional problems in GP-SR, a SHAP-based feature selection method was also adopted. Combining machine learning models and GP-SR, design equations for the characteristic points for the backbone curve of RC walls were proposed. The principal conclusions can be summarized as follows:

1. To predict the characteristic points for the backbone curve of RC walls, five machine learning models were applied to 388 existing RC walls with various failure modes. Among the five machine learning models, XGBoost exhibited the best prediction performance, and the tree based models in DT, RF, and XGBoost showed better prediction performance than SVM and ANN.
2. SHAP analysis results indicate that the relationships between the input and output variables are affected by failure modes (or failure mechanisms). Thus, it is necessary to model the backbone curves of RC walls individually, addressing each failure mode.
3. The backbone curves of RC walls with various failure modes were modelled separately based on GP-SR. The input variables to train models were filtered by SHAP analysis. The prediction result in test set indicates that proposed model predicts well the test results of RC walls.
4. The proposed GP-SR model and ASCE 41-17 were applied to the existing test results for comparison. The results showed that the GP-SR model predicted the characteristic points for the backbone curve better than ASCE 41-17. It is noted that as the proposed model is affected by failure modes, the failure mode of RC walls should be judged to estimate the backbone curve.

**Acknowledgements**

This research was sponsored by the National Natural Science Foundation of China (Grant No. 51878268), the Huxiang Youth Talent Support Program of Hunan Province, China (2021RC3041), the Natural Science Foundation of Hunan Province, China (Grant No. 2020JJ4195), the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No.2021R1A4A3030117), and the Korea Agency for Infrastructure Technology Advancement (KAIA) funded by the Ministry of Land, Infrastructure and Transport (Grant RS-2022-00143493).

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